1. Let $A=\left[a_{ij}\right]$ be a square matrix of order 3 and B=$\left[b_{ij}\right]$ be a matrix such that $b_{ij}=2^{i-j}a_{ij}$ for $1\le i,j\le 3,\forall i,j\in N.$ If the determinant of A is same as its order, then the value of $\left|\left(B^{T}\right)^{-1}\right|$ is

A) $\frac{1}{3}$ B) 3 C) 9 D) $\frac{1}{27}$

2. A person predicts the outcome of 20 cricket matches of his home team. Each match can result either in a win, loss or tie for the home team. The total number of ways in which he can make the predictions such that exactly 10 predictions are correct are equal to

A) ${}^{20}c_{10}.2^{10}$ B) ${}^{20}c_{10}.3^{20}$ C) ${}^{20}c_{10}.3^{10}$ D) ${}^{20}c_{10}.2^{20}$

3. The quadratic equation whose roots are the arithmetic mean and the harmonic mean of the roots of the equation $x^2+7x-1=0$ is

A) $14x^2+14x-45=0$ B) $45x^2-14x+14=0$

C) $14x^2+45x-14=0$ D) $45x^2+14x-45=0$

4. If a+b+c=3 (where a,b,c>0) , then the greatest value of $a^2b^2c^2$ is

A) $\frac{3^{10}2^4}{7^7}$ B) $\frac{3^9 2^4}{7^7}$ C) $\frac{3^8 2^4}{7^7}$ D) $\frac{3^9 2^3}{7^6}$

5. The point at which the line segment joining A(1,1) and B(5,5) subtends on obtuse angle is

A) (7,7) B) (0,5) C) (2,4) D) (1,5)

6. The average weight of students in a class of 35 students is 40 kg. If the weight of the teacher is included, then the average rises by $\frac{1}{3}$ kg. The weight of the teacher is

A) 40.5 kg B) 50 kg C) 41 kg D) 52 kg

7. Two straight roads OA and OB intersect at O. A tower is situated with in the angle formed by them and subtends angle of $45^0\ \&\ 30^0$ at the point A and B, where the roads are nearest to it. If OA=400 meters and OB=300 meters, then the height of the tower is

A) $250\sqrt{2}$ meters B) 500 meters C) $50\sqrt{14}$ meters D) $100\sqrt{7}$ meters

8. The value of $\lim\limits_{x\to-\infty}\frac{x^2\tan\left(\frac{1}{x}\right)}{\sqrt{4x^2-x+1}}$ is equal to

A) 1 B) $\frac{1}{2}$ C) -1 D) -$\frac{1}{2}$

9. The range of the function $f\left(x\right)=\sin^{-1}\left[x^2-\frac{1}{3}\right]-\cos^{-1}\left[x^2+\frac{2}{3}\right]$ is (where [x] represents the greatest integer value of x)

A) $\left[-\pi,0\right]$ B) $\left\{-\pi,0\right\}$ C) $\left\{0,\pi\right\}$ D) $\left\{0,\pi,-\pi\right\}$

10. The point on the curve $6y=4x^3-3x^2$, the tangent at which makes on equal angle with the coordinate axes is

A) $\left(1,\frac{-1}{6}\right)$ B) $\left(-1,\frac{-7}{6}\right)$ C) $\left(\frac{-1}{2},\frac{-5}{24}\right)$ D) $\left(\frac{1}{2},\frac{-1}{24}\right)$

11. Let $\int \frac{dx}{\sqrt{x^2+1}-x} = f(x)+c$ such that $f(0)=0$ and c is the constant of integration, then value of $f(1)$ is

A) $\frac{1}{\sqrt{2}}+\frac{1}{2}\ln\left(1+\sqrt{2}\right)$ B) $\frac{1}{2}+\frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$ C) $\frac{1}{2}+\frac{1}{2}\ln\left(1+\sqrt{2}\right)$ D) $\frac{1}{\sqrt{2}}+\frac{1}{2}\left(1+\ln\left(1+\sqrt{2}\right)\right)$

12. The solution of differential equation $x\frac{dy}{dx}=y\log\left(\frac{y^2}{x^2}\right)$ is (where c is an arbitrary constant)

A) $y=x.e^{cx+1}$ B) $y=x.e^{cx-1}$ C) $y=x^2.e^{cx+1}$ D) $y=x.e^{cx^2+\frac{1}{2}}$

SOLUTIONS

MATHEMATICS

1. $\text{given}|A|=3\ \&\ B=\begin{bmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33} \end{bmatrix} \Rightarrow |B|=3$

$\therefore \left|\left(B^T\right)^{-1}\right| = \frac{1}{B^T} = \frac{1}{|B|} = \frac{1}{3}$

2. In order to predict exactly 10 correct predictions

$\Rightarrow$we have to select 10 matches out of 20 in ${}^{20}c_{10}$ ways

$\Rightarrow$remaining 10 predictions must be wrong, so for each incorrect prediction, we have 2 choices out of 3, this can be done in 2^{10}

$\therefore$ required no. of ways ${}^{20}c_{10}.2^{10}$

3. $\alpha+\beta=-7, \alpha\beta=-1$

Now, A.M= $\frac{\alpha+\beta}{2}=\frac{-7}{2}$ & H.M = $\frac{2\alpha\beta}{\alpha+\beta}=\frac{2}{7}$

Required equation is $x^2-\left(\frac{-7}{2}+\frac{2}{7}\right)x+\left(-\frac{7}{2}\right)\left(\frac{2}{7}\right)=0$

$\Rightarrow 14x^2+45x-14=0$

4. Given a+b+c=3 $\Rightarrow 2.\frac{a}{2}+3.\frac{b}{3}+2.\frac{c}{2}=3$

Now, A.M $\geq$ G.M

$$\Rightarrow \frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7}\geq\left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^2\left(\frac{c}{2}\right)^2\right)^{\frac{1}{7}}$$

$$\Rightarrow \frac{a+b+c}{7}\geq\left(\frac{a^2b^3c^2}{2^4.3^3}\right)^{\frac{1}{7}}\Rightarrow a^2b^3c^2=\frac{2^4.3^{10}}{7^7}$$

5. 4) (2,4)

Equation of the circle assuming AB as the diameter is $(x-1)(x-5)+(y-1)(y-5)=0$

$\Rightarrow x^2+y^2-6x-6y+10=0$

Now, for point (h,k) and circle s=0

If $s_1<0$ then the point (h.k) lies inside the circle s=0 and at this point diameter of the circle subtends an obtuse angle

Now, from options, we get

(7-1)(7-5)+(7-1)(7-5)>0

(0-1)(0-5)+(5-1)(5-5)>0

(2-1)(2-5)+(4-1)(4-5)<0

(1-1)(1-5)+(5-1)(5-1)=0

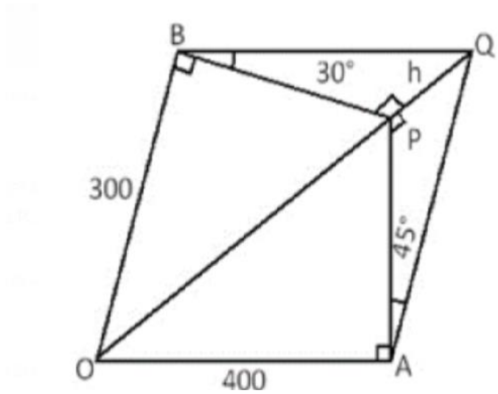
Hence, (2,4) is the required point.

6. Let , the weight of the teacher be w kg,

Then $40+\frac{1}{3}=\frac{35\times40+w}{35+1}$

$\Rightarrow w=52$

7.



$PAQ = 45^0$ & $PBQ = 30^0$

$\Rightarrow \frac{PA}{h} = \cos 45^0$ & $\frac{PB}{h} = \cot 30^0$

$\Rightarrow h = PA$ & $PB = \sqrt{3}h$

Now, $OP^2 = PA^2 + OA^2 = PB^2 + OB^2$

$\Rightarrow h^2 + (400)^2 = 3h^2 + (300)^2 \Rightarrow h = 50\sqrt{14}$ meters

8. 1) $-\frac{1}{2}$

$$\underset{x\to-\infty}{Lt} \frac{\frac{\tan\frac{1}{x}}{\frac{1}{x}}}{-x\sqrt{4-\frac{1}{x}+\frac{1}{x^2}}} = \frac{1}{-\sqrt{4}} = \frac{-1}{2}$$

9. $\left[x^2 - \frac{1}{3}\right] = \left[x^2 + \frac{2}{3} - 1\right] = \left[x^2 + \frac{2}{3}\right] - 1 = k$ (say)

The function is $\sin^{-1} k - \cos^{-1}(k+1)$

For the above function to be defined, $-1 \le k \le 1$ & $-1 \le (k+1) \le 1$

$\therefore k = \{-1, 0\}$

$\therefore$ range $= \left\{\sin^{-1}(1) - \cos^{-1}(0), \sin^{-1}(0) - \cos^{-1}(1)\right\} = \{-\pi, 0\}$

10. 3) $\left(\frac{-1}{2},\frac{-5}{24}\right)$

$$6y=4x^3-3x^2$$

$$6\frac{dy}{dx}=12x^2-6x$$

$$\frac{dy}{dx}=2x^2-x=\pm 1$$

$$2x^2-x-1=0 \text{ or } 2x^2-x+1=0$$

$$(x-1)(2x+1)=0 \text{ or } x\in\theta$$

$$x=1, x=-\frac{1}{2}$$

Hence points are $\left(1,\frac{1}{6}\right)$ and $\left(-\frac{1}{2},\frac{-5}{24}\right)$

11. 4) $\frac{1}{\sqrt{2}}+\frac{1}{2}\left(1+\log\left(1+\sqrt{2}\right)\right)$

$$\int\frac{\sqrt{x^2+1}+x}{1}dx$$

$$\int\sqrt{x^2+1}dx+\int xdx$$

$$=\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\log\left|x+\sqrt{x^2+1}\right|+\frac{x^2}{2}+c$$

$$f(x)=\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\ln\left|x+\sqrt{x^2+1}\right|+\frac{x^2}{2}$$

$$f(0)=0$$

$$f(2)=\frac{\sqrt{2}}{2}+\frac{1}{2}\log\left(1+\sqrt{2}\right)+\frac{1}{2}$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{2}\left(1+\log\left(1+\sqrt{2}\right)\right)$$

12. 4) $xe^{ex^2+\frac{1}{2}}$

Putting $y = xv$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 2v\log v$

$\int \frac{dx}{x} = \int \frac{dv}{v\left(2\ln v - 1\right)}$

On integrating we get

$y = xe^{ec^2+\frac{1}{2}}$